Weibel instability due to inverse bremsstrahlung absorption

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A new Weibel source due to the inverse bremsstrahlung absorption is presented. It has been shown that in homogeneous plasmas, this mechanism may drive strong collisionless Weibel modes with growth rates of order of $\gamma\sim 10^{11}\ \mathrm{s}^{-1}$ and negligible group velocities. In the laser-produced plasmas, for short laser wavelengths ($\lambda_L<1\ \mu\mathrm{m}$) and high laser fluxes ($I>10^{14}\ \mathrm{W/cm}^2$), this Weibel source is most efficient as the ones due to the heat flux and the plasma expansion. The useful scaling law of the convective e-foldings, with respect to the laser and the plasma parameters, is also derived. [S1063-651X(97)07206-1]

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I. INTRODUCTION

The presence of strong magnetic fields (in a megagauss range) in laser irradiated targets could be detrimental to the process of ablative implosion, necessary for achieving thermonuclear fusion reactions. Indeed, several effects could be induced by these fields, such as the anomalous reduction of the electron heat flux from the laser energy deposition layer to the ablation surface, reduction of the mass ablation rate, filamentation of the plasma flow, etc. Various mechanisms responsible for producing such B fields have been reported in the literature: thermoelectric effects [1], nonlinear effects [2], Rayleigh-Taylor [3] and electromagnetic [4] instabilities, etc. In this paper, we deal with the Weibel instability due to the inverse bremsstrahlung absorption in homogeneous plasmas and laser-produced plasmas. It has been shown by Weibel [4] that anisotropic distribution functions in the velocity space may drive unstable electromagnetic modes. For a symmetrical angular distribution function about the x axis $f(v_x,v,x)$, a positive second anisotropic distribution function ($T_x>T_\perp$) drives unstable $k_\perp$ modes, whereas a negative second anisotropic distribution function ($T_x<T_\perp$) drives unstable k_x modes [5]. Here, the subscripts x and $\perp$, denote the parallel and perpendicular direction to the x axis. These k_x and $k_\perp$ modes were extensively studied in the overdense plasma by Epperlein and co-workers [6] in the collisional limit ($k\lambda\ll 1$) and by Ramani and Laval [7] in the collisionless one ($k\lambda>1$); k being the wave number and $\lambda(n_e,T)=4\pi\epsilon_0^2T^2/n_ee^4(Z+1)\ln\Lambda$, the electron mean free path, where T denotes the electron temperature, n_e the electron density, e the electron charge, Z the ion charge number, and $\ln\Lambda$ the Coulomb logarithm. In these works, it has been shown that in the conduction region, the k_x mode is stable whereas the $k_\perp$ mode is moderately unstable (the growth rate is of the order of $\gamma\sim 10^9\ \mathrm{s}^{-1}$). On the other hand, using Fokker-Planck simulations, Matte, Bendib and Luciani [8] have been pointed out strongly unstable collisionless k_x modes ($\gamma\sim 10^{11}\ \mathrm{s}^{-1}$) in the underdense plasma.

In this paper, we present a first analytic analysis of Weibel modes due to the inverse bremsstrahlung absorption (IBA) source. The Weibel source has been computed through an analytic model based on the plasma kinetic theory. The physical mechanism is fairly straightforward: the IBA of the laser pulse is produced preferentially along the electric-field direction, resulting in a weak plasma temperature anisotropy. So, for a circularly polarized laser wave, $\mathbf{E}=E(\mathbf{y}+i\mathbf{z})$, it results to a temperature anisotropy, $T_x<T_\perp$, which drives unstable k_x modes, whereas for a linearly polarized laser wave, $\mathbf{E}=E\mathbf{x}$, it results to a temperature anisotropy, $T_x>T_\perp$, which drives unstable $k_\perp$ modes.

This work is organized as follows. In Sec. II the basic kinetic equation and the semicollisional dispersion relation of quasistatic electromagnetic waves are presented. Section III is devoted to the stability analysis of the Weibel k_x modes. For this, we compute explicitly the Fokker-Planck equation in presence of a high frequency electric field and deduce the group velocity, the growth rate, and the number of convective e-foldings. The useful expression of the number of convective e-foldings with respect to the plasma and the laser parameters is also derived. Finally, a discussion and a conclusion are given in Sec. IV.

II. BASIC EQUATIONS

Throughout this work we use the Fokker-Planck (FP) equation which describes, in particular, the thermal transport and the light energy absorption in fully ionized plasmas. Following the Braginskii [9] notations, the FP equation for the electrons reads:

$$\frac{\partial f}{\partial t}+\mathbf{v}\cdot\frac{\partial f}{\partial \mathbf{r}}-\frac{e}{m_e}(\mathbf{E}+\mathbf{v}\times\mathbf{B})\cdot\frac{\partial f}{\partial \mathbf{v}}=C_{\mathrm{ei}}(f), \qquad (1)$$

where

$$C_{\mathrm{ei}}(f)=\frac{\nu}{v^3}\frac{\partial}{\partial v_i}(v_iv_j-v^2\delta_{ij})\frac{\partial f}{\partial v_j}. \qquad (2)$$

In Eqs. (1) and (2), $\mathbf{E}$ and $\mathbf{B}$ are the electric and magnetic fields respectively, $\nu=v_t^4/2\lambda$, where $v_t=\sqrt{T/m_e}$ is the electron-thermal velocity. The right-hand side (RHS) of Eq. (1) corresponds to the collision terms, where C_{ei} is the Landau electron-ion collision operator. We have neglected in Eq.

(1), the electron-electron collision term, which corresponds to the Lorentz plasma approximation (high Z limit) and in Eq. (2), the terms due to the energy exchange between ions and electrons, of the order of m_e/m_i.

For the Weibel modes analysis, Eq. (1) should be coupled with the semicollisional dispersion relation derived in Ref. [5]. This dispersion relation is valid in the whole collisionality regime and is derived in the local approximation, $kL \gg 1$; L being the plasma inhomogeneity scale length and the Lorentz gas approximation. Practical expressions of the group velocity V_g and the growth rate γ, using numerical fits of the continuous fractions with a precision better than 5%, are computed. For the k_x mode, these expressions are given by

$$V_g = \left[\frac{\sqrt{6}}{2} v_t \int_0^\infty y^{5/2} F(y,k\lambda) f_1(y)\, dy \right] \Big/ D \tag{3}$$

and

$$\gamma(k) = \left[\frac{3}{64\pi} \frac{n_e}{\lambda v_t^2} \frac{k^2 c^2}{\omega_p^2} + \frac{3\sqrt{2^5}}{\sqrt{5}} \lambda v_t k^2 \int_0^\infty y^{9/2} G(y,k\lambda) f_2(y)\, dy \right] \Big/ D, \tag{4}$$

where,

$$D = \int_0^\infty y^3 F(y,k\lambda) \frac{\partial f_0}{\partial y}\, dy,$$

$$F = [(1+(a/\delta)^2)^{-1/2}]/2,$$

$$G = 2(1+a^2\theta)/[3(1+a^2\beta)(1+2F)],$$

$$a = 8k\lambda y^2, \quad \delta = 3\pi/2, \quad \theta = 30\beta/\delta^2$$

$$\text{and} \quad \beta = (5\delta^2/252 - 3/4)/(\delta^2 - 30). \tag{5}$$

In Eqs. (3)–(5), c is the light speed, ω_p is the plasma frequency, $y = v^2/2v_t^2$ is the normalized square velocity, f_0 is the isotropic distribution function, and f_1, f_2, the first and the second anisotropic distribution function defined through the expansion of the distribution function upon the Legendre polynomials, $P_l(v_x/v)$,

$$f = \sum_{l=0}^{\infty} f_l(v) P_l(v_x/v) \sqrt{2l+1}. \tag{6}$$

Equation (3) shows that the group velocity is described by the first anisotropic distribution function f_1, whereas in Eq. (4), the first term represents the dissipative effects and the second one, the Weibel source, which depends on the second anisotropy f_2. Note that, Eqs. (3) and (4) recover in the collisionless limit $(k\lambda > 1)$, the results derived in Ref. [7]. For the Weibel instability analysis in inhomogeneous plasmas, the relevant physical quantity is the number of convective e-foldings C. This quantity may be computed with the WKB method

$$C(k_i) = \int_{x_i}^{x_f} \gamma[k(k_i,x),x]/V_g[k(k_i,x),x]\, dx, \tag{7}$$

where k_i is the optimum wave number of the Weibel mode at the plasma layer defined by the density $n_e(x_i)$ and $(x_f - x_i)$ is the extent of the growth region. The wave number $k = k(k_i,x)$ varies so as to keep the real part of the frequency ω_r, fixed.

III. WEIBEL SOURCE DUE TO INVERSE BREMSSTRAHLUNG ABSORPTION

A. Fokker-Planck equation in presence of a high-frequency electric field

Let us now compute the electron distribution function of an unmagnetized plasma in the presence of an oscillating electric field. As we aim at obtaining in this section the IBA contribution to the Weibel source, we consider for simplicity, homogeneous plasmas in order to avoid the Weibel sources due to the thermal transport and the plasma expansion [10]. In order to take into account the high-frequency (hf) response of plasma electrons to the laser field excitation, we split up the distribution function into a low-frequency (lf) part f_s and a part f_h, which oscillates at the laser frequency ω_0. From Eq. (1), we deduce the hf and the lf equations

$$\frac{\partial f_s}{\partial t} - \frac{e}{m_e} \left\langle \mathbf{E}_h \cdot \frac{\partial f_h}{\partial \mathbf{v}} \right\rangle = C_{\mathrm{ei}}(f_s), \tag{8}$$

$$\frac{\partial f_h}{\partial t} - \frac{e}{m_e} \mathbf{E}_h \cdot \frac{\partial f_s}{\partial \mathbf{v}} = C_{\mathrm{ei}}(f_h), \tag{9}$$

where the bracket $\langle \dots \rangle$ denotes the average over the laser period. The explicit derivation of Eqs. (8) and (9) needs to specify the laser-wave polarization.

(i) Linearly polarized laser wave. The laser electric field can be expressed as

$$\mathbf{E}_h = R_e[E_0 \exp(i\omega_0 t)]\mathbf{x} \tag{10}$$

and the distribution function can be expanded as

$$f(\mathbf{v},t) = f_s(v,\mu,t) + \mathrm{Re}[f_h(v,\mu)\exp(i\omega_0 t)], \tag{11}$$

where $\mu = \cos\theta = v_x/v$.
Using Eqs. (2), (10), and (11), Eqs. (8) and (9) become

$$\frac{\partial f_s}{\partial t} - \frac{\nu}{v^3} \frac{\partial}{\partial \mu} (1-\mu^2) \frac{\partial f_s}{\partial \mu} = \frac{1}{2} \mathrm{Re}\left[\frac{eE_0^*}{m_e} \left(\mu \frac{\partial f_h}{\partial v} + \frac{1}{v}(1-\mu^2) \frac{\partial f_h}{\partial \mu} \right) \right] \tag{12}$$

and

$$i\omega_0 f_h - \frac{\nu}{v^3}\frac{\partial}{\partial\mu}(1-\mu^2)\frac{\partial f_h}{\partial\mu} = \frac{eE_0}{m_e}\left[\mu\frac{\partial f_s}{\partial v} + \frac{1}{v}(1-\mu^2)\frac{\partial f_s}{\partial\mu}\right], \tag{13}$$

where the notation (*) denotes conjugate complex.

For solving Eqs. (12) and (13), we use the following expansions

$$f_s = \sum_{l=0}^{\infty}\sqrt{2l+1}P_l(\mu)f_{s_l}(v)$$

$$\text{and}\quad f_h = \sum_{l=0}^{\infty}\sqrt{2l+1}P_l(\mu)f_{h_l}(v),$$

where we recall that the $P_l(\mu)$ are the Legendre polynomials. Using recursion relations [11] of the Legendre polynomials, after some algebra, Eq. (13) becomes

$$\overline{f_{h_l}}(v) = \left[-\frac{ieE_0}{\omega_0 m_e} + \frac{eE_0}{\omega_0^2 m_e}\frac{l(l+1)\nu}{v^3}\right]\left[\frac{lv^{l-1}}{2l-1}\frac{\partial}{\partial v}\frac{\overline{f_{s_{l-1}}}}{v^{l-1}} + \frac{2l+1}{2l+3}\frac{1}{v^{l+2}}\frac{\partial}{\partial v}v^{l+2}\overline{f_{s_{l+1}}}\right], \tag{14}$$

where the high-frequency approximation [12] $\omega_0 v_t^3 \gg \nu$, and the notations $\overline{f_{s_l,h_l}} = \sqrt{2l+1}f_{s_l,h_l}$, have been used. Substituting the relation (14) into Eq. (12), we obtain the secular FP equation

$$\frac{\partial\overline{f_{s_l}}}{\partial t} + \frac{\nu}{v^3}l(l+1)\overline{f_{s_l}} = \frac{\nu v_0^2}{2}\left[\frac{l^2(l-1)}{2l-1}v^{l-1}\frac{\partial}{\partial v}\left(\frac{l-1}{2l-3}\frac{1}{v^4}\frac{\partial}{\partial v}\frac{\overline{f_{s_{l-2}}}}{v^{l-2}} + \frac{l}{2l+1}\frac{1}{v^{2l+3}}\frac{\partial}{\partial v}v^{l+1}\overline{f_{s_l}}\right)\right] + \frac{\nu v_0^2}{2}\left[\frac{(l+1)^2(l+2)}{2l+3}\times\frac{1}{v^{l+2}}\frac{\partial}{\partial v}\left(\frac{l+1}{2l+1}v^{2l-1}\frac{\partial}{\partial v}\frac{\overline{f_{s_l}}}{v^l} + \frac{l+2}{2l+5}\frac{1}{v^4}\frac{\partial}{\partial v}v^{l+3}\overline{f_{s_{l+2}}}\right)\right], \tag{15}$$

where $v_0 = |eE_0/m_e\omega_0|$, is the quiver velocity of oscillation of the electron in the hf electric field.

(ii) Circularly polarized laser wave. Similarly, for a circularly polarized hf laser field,

$$\mathbf{E}_h = \frac{1}{\sqrt{2}}R_e[E_0\exp(i\omega_0 t)(\hat{\mathbf{y}} + i\hat{\mathbf{z}})]. \tag{16}$$

The distribution function can be split up into

$$f(\mathbf{v},t) = f_s(v,\mu) + \mathrm{Re}[f_h(v,\mu)\exp(i\omega_0 t + i\phi)], \tag{17}$$

where $\phi = \mathrm{arctg}(v_z/v_y)$. From Eqs. (2), (8), (9), (16), and (17), we deduce the lf and the hf equations,

$$\frac{\partial f_s}{\partial t} - \frac{\nu}{v^3}\frac{\partial}{\partial\mu}(1-\mu^2)\frac{\partial f_s}{\partial\mu} = \frac{1}{2}\mathrm{Re}\left[\frac{eE_0^*}{m_e\sqrt{2}}\left(\sin(\theta)\left(\frac{\partial f_h}{\partial v} - \frac{\mu}{v}\frac{\partial f_h}{\partial\mu}\right) + \frac{1}{\sin\theta}\frac{f_h}{v}\right], \tag{18}$$

$$i\omega_0\overline{f_h} - \frac{\nu}{v^3}\frac{\partial}{\partial\mu}(1-\mu^2)\frac{\partial\overline{f_h}}{\partial\mu} = \frac{eE_0}{m_e\sqrt{2}}\sin(\theta)\left[\frac{\partial\overline{f_s}}{\partial v} - \frac{\mu}{v}\frac{\partial\overline{f_s}}{\partial\mu}\right]. \tag{19}$$

In this case we use the expansions

$$f_s = \sum_{l=0}^{\infty}\sqrt{2l+1}P_l(\mu)f_{s_l}(v), \tag{20}$$

$$f_h = \sum_{l=0}^{\infty}\sqrt{2l+1}P_l^1(\mu)f_{h_l}(v), \tag{21}$$

where the $P_l^1(\mu)$ are the associated Legendre functions. These expansions are imposed by the geometry of the problem. Using Eqs. (18)–(21), and recursion relations [11] on the $P_l^1(\mu)$, we obtain

$$\overline{f_{h_l}}(v) = \left[-\frac{ieE_0}{\omega_0 m_e\sqrt{2}} + \frac{eE_0}{\omega_0^2 m_e\sqrt{2}}\frac{l(l+1)\nu}{v^3}\right]\times\left[\frac{1}{2l-1}\left(\frac{\partial\overline{f_{s_l}}}{\partial v} - (l-1)\frac{\overline{f_{s_l}}}{v}\right) - \frac{1}{2l+3}\left(\frac{\partial\overline{f_{s_l}}}{\partial v} + (l+2)\frac{\overline{f_{s_l}}}{v}\right)\right], \tag{22}$$

$$\frac{\partial\overline{f_{s_l}}}{\partial t} + \frac{\nu}{v^3}l(l+1)\overline{f_{s_l}} = \frac{\nu v_0^2}{4}\left[\frac{(l+1)^2(l+2)^2}{2l+3}\frac{1}{v^{l+2}}\frac{\partial}{\partial v}\left(\frac{1}{2l+1}v^{2l-1}\frac{\partial}{\partial v}\frac{\overline{f_{s_l}}}{v^l} - \frac{1}{2l+5}\frac{1}{v^4}\frac{\partial}{\partial v}v^{l+3}\overline{f_{s_{l+2}}}\right)\right] - \frac{\nu v_0^2}{4}\times\left[\frac{l^2(l-1)^2}{2l-1}v^{l-1}\frac{\partial}{\partial v}\left(\frac{1}{2l-3}\frac{1}{v^4}\frac{\partial}{\partial v}\frac{\overline{f_{s_{l-2}}}}{v^{l-2}} - \frac{1}{2l+1}\frac{1}{v^{2l+3}}\frac{\partial}{\partial v}v^{l+1}\overline{f_{s_l}}\right)\right]. \tag{23}$$

Note that Eqs. (15) and (23), recover for $l=0$, the isotropic equation derived by Langdon [12], and give more generalized results, which take into account the contribution of the IBA to the anisotropic distribution functions f_{s_l} $(l>0)$.

B. Group velocity, growth rate, and number of convective *e*-foldings

Keeping in Eqs. (15) and (23) the dominant terms corresponding to the lower order with respect to the small param-

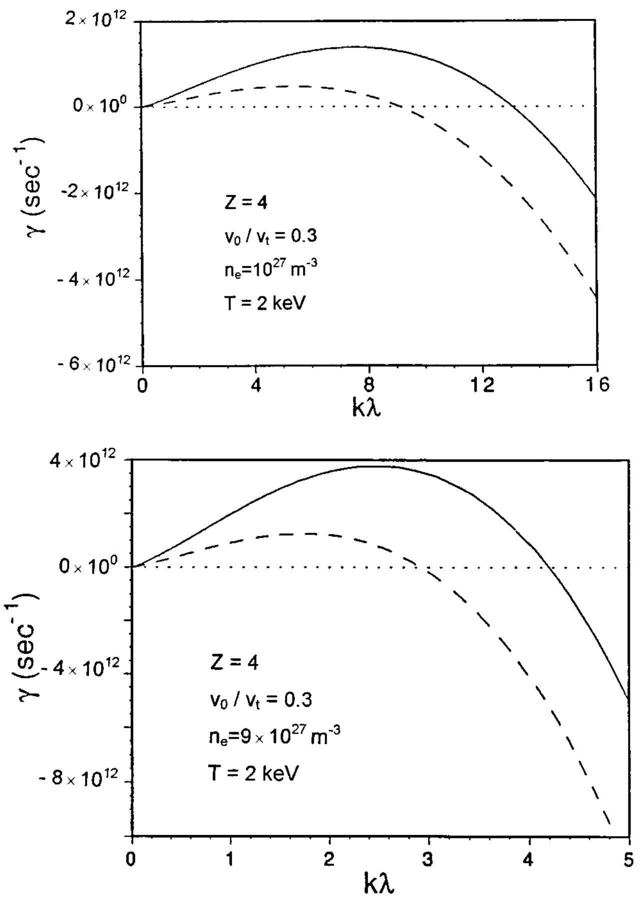

FIG. 1. Maximum growth rate $\gamma_{\max}$ versus the parameter $k\lambda$ (k is the wave number and λ the electron mean free path) for typical laser and plasma parameters. The solid line and the dashed line correspond, respectively, to the linearly and circularly laser wave.

eter $v_0/v_t \sim \epsilon$, it results in the quasistatic approximation ($\partial f/\partial t \approx 0$), the second distribution function equation

$$\frac{1}{3\sqrt{5}}\, p\nu v_0^2\left[v\,\frac{\partial}{\partial v}\left(\frac{1}{v^4}\,\frac{\partial}{\partial v}\,f_{s0}\right)\right]-\frac{6\nu}{v^3}\,f_{s2}=0, \qquad (24)$$

where $p=-1$, 2 for a circularly and a linearly polarized laser wave, respectively. We can see from Eqs. (15) and (23), that the first distribution function and more generally the odd order components of the distribution function are negligible, i.e., $f_{2n+1}\sim 0$, and on the other hand, from Eq. (24), that the second distribution function scales as $(v_0/v_t)^2 \sim \epsilon^2$. We can conclude that the IBA source does not contribute efficiently to the convection of quasistatic magnetic structures and it may be an efficient mechanism for their amplification, since, as it is known [5,7], a second-order ($\sim\epsilon^2$) anisotropic distribution function f_{s2} corresponds, in Eq. (4), to a strong Weibel source term. We give in Fig. 1, the spectrum of the growth rate for two typical numerical cases. As expected, very high growth rates have been obtained ($\gamma > 10^{11}$ s^{-1}). We note that the most unstable modes range in the collisionless regime and that in the k space, the growth region is shifted to the low $k\lambda$ values for decreasing laser wavelength λ_L. In such homogeneous plasmas, the magnetic modes can grow strongly and stagnate instead of being convected away as it is shown in Refs. [5–8]. It results that these Weibel modes may reach huge intensities even for localized IBA of the laser energy.

In inhomogeneous plasmas, one has to take into account both gradient and IBA sources. In order to get an estimate of the IBA to the Weibel instability in laser-created plasmas, one neglects the contributions of the heat flux and the plasma expansion [10] sources to the second anisotropic distribution function and keeps the contribution of the heat flux source to the first anisotropic distribution function. For simplicity, we assume a gentle inhomogeneous steady-state plasma, thus, the temperature anisotropy is weak and in Eq. (24), f_{s0} is assumed to be a Maxwellian. Furthermore, we consider a laser wave normally incident on a plasma defined by an isothermal corona and a linear density profile, i.e., $n_e = n_c\, x/L_n$. The laser electric field E_h is computed numerically from the wave equation [13]

$$\frac{\partial^2 \mathbf{E}_h}{\partial x^2}+\frac{\omega_0^2}{c^2}\,\mathbf{E}_h-\mu_0\,\frac{\partial \overline{J_e}}{\partial t}=\mathbf{0}, \qquad (25)$$

and the fluid electron motion equation,

$$\frac{\partial \mathbf{V}_e}{\partial t}+\frac{\nu_t}{\lambda(n(x),T)}\,\mathbf{V}_e=-\frac{e\mathbf{E}_h}{m_e}, \qquad (26)$$

where $\mathbf{J}_e=-en_e\mathbf{V}_e$ is the electron current density and $\mathbf{V}_e$, the fluid electron velocity. Using a circularly polarized laser field, we have computed from Eq. (7), the number of convective e-foldings for the k_x modes. Three numerical cases have been performed

case 1: $\lambda_L=1.06$ μm, $\tau_L=1$ ns, $I_a=10^{14}$ W/cm^2, $Z=A/2=4$

case 2: $\lambda_L=0.53$ μm, $\tau_L=0.6$ ns, $I_a=7\times10^{14}$ W/cm^2, $Z=A/2=4$

case 3: $\lambda_L=0.353$ μm, $\tau_L=0.6$ ns, $I_a=2\times10^{15}$ W/cm^2, $Z=A/2=4$,

where, λ_L is the laser wavelength, τ_L is the pulse duration, I_a is the absorbed laser intensity, and Z is the ion charge number. The extents of the growth region are $(x_f-x_i) \approx 0.14L_n$, $0.12L_n$, and $0.10L_n$ and the numbers of convective e-foldings are $C_{\rm IBA}=115$, 92, and 47, respectively. These results show that the IBA source may drive strong unstable magnetic modes at the vicinity of the critical layer. For practical purposes, we have computed numerically the expression of $C_{\rm IBA}$ with respect to the relevant plasma and laser parameters

$$C_{\rm IBA}=0.13\left(\frac{\gamma_{\max}(\mathrm{s}^{-1})}{10^{11}\ \mathrm{s}^{-1}}\right)\left(\frac{10^5\ \mathrm{m/s}}{V_g(\mathrm{m/s})}\right)L_n(\mu\mathrm{m}) \times\left(1-6.3\times10^{-4}\,\frac{L_n}{\lambda_L}\right), \qquad (27)$$

where

$$V_g(\text{m/s})=1.84\times10^5\left(\frac{Z+1}{Z}\right)^{1/2}\left(\frac{I_a}{10^{14}\ \text{W/cm}^2}\right)^{1/3}\left(\frac{\lambda_L}{1\ \mu\text{m}}\right)^{2/3},$$

and

$$\gamma_{\max}(s^{-1})=7.2\times10^9\left(\frac{Z+1}{Z}\right)\times\left(\frac{I_a}{10^{14}\ \text{W/cm}^2}\right)^{5/63}\left(\frac{\lambda_L}{1\ \mu\text{m}}\right)^{2/3}\left(\frac{L_n}{\lambda_L}\right)^{1/2}\times\left(\frac{2-A_{\text{IBA}}+2(1-A_{\text{IBA}})^{1/2}}{A_{\text{IBA}}}\right)^{3/2}\times\exp\left(-0.69\,\frac{v_t}{c}\,\frac{L_n}{\lambda_L}\right).$$

The scaling law of the electron temperature is defined in Ref. [14] and the density scale length is roughly $L_n \approx c_s\tau_L$. The IBA rate for a linear density profile is given by [13]

$$A_{\text{IBA}}=1-\exp\left(\frac{32}{15}\,\frac{v_t}{c}\,\frac{L_n}{\lambda_L}\right).$$

We have checked for several numerical cases that Eq. (27) gives numerical fits with a good precision ($\leqslant 7\%$).

IV. DISCUSSION AND CONCLUSION

We have shown in this work that a high-frequency electric field with moderate strength may constitute an efficient Weibel source in homogeneous plasmas. The growth rate of the Weibel modes rate may reach $10^{11}\ \text{s}^{-1}$ and the group velocity is negligible. In laser-created plasmas the IBA mechanism competes with the plasma expansion (PE) and the heat flux (HF) mechanisms [10]. In any case, the unstable Weibel modes are convectively amplified towards the underdense plasma and the group velocity is comparable to the ion acoustic speed, $V_g \sim c_s$. The magnitude and the spatial location of the magnetic field strongly depend on the laser wavelength. For large laser wavelengths ($\lambda_L \sim 10\ \mu$m), the generation of the magnetic field due to the IBA source in the vicinity of the critical layer, is as efficient as the ones due to the HF and the PE sources [10]. For short laser wavelengths, ($\lambda_L \sim 1\ \mu$m), the growth region of the B field due to the PE source is located on the very underdense plasma, whereas, the HF and the IBA sources are confined to the vicinity of the critical layer where, the thermal gradients are important and, the laser energy is deposited. For decreasing laser wavelengths, the number of convective e-foldings strongly decreases for the HF source [10] ($C_{\text{hf}} \sim \lambda_L^{11/3}$), whereas it is still important for the IBA source. In conclusion, strong collisionless magnetic modes may be driven by the Weibel mechanisms in the corona of laser-created plasmas. For a spatial amplification factor, $\exp(C) \sim 10^3$, corresponding to $C \sim 7$, a seed of magnetic field of few kilogauss in the vicinity of the critical layer, may be convectively amplified to the megagauss range. In this range of intensity the nonlinear saturation, as the mode coupling, occurs and rigorously our theory is no longer valid. These fields may inhibit the thermal transport, could impede the expansion of the corona, and may contribute to the production of filamentary structures. For future laser-produced plasma experiments, corresponding to long laser duration and short laser wavelength, the plasma gradients are weak and the IBA source should be the most efficient mechanism for producing such megagauss B fields.

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